MODEL OF THE MOTION OF FISHES AND SNAKES

M. A. Lavrent'ev

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In June of the present year, the author has published, under the above title, a note in the Proceedings of the French Academy. The note included a qualitative solution of the following problem: Is it possible to obtain in an ideal liquid the schematic motion of a body (without formation of eddies) only by internal forces generated in the body ("muscle" tension inside the body, no volume changes)?

A rectangular plate of length l, thickness δ, and width 1, with $\delta \leq 1 \leq l$, was used as model of the moving body.

It was assumed during the time of the motion that the plate could bend according to some law but remained cylindrical in the bending process. This model was used to determine the motions of the plate for which the plate could move in the liquid. A more detailed analysis of this motion has revealed that the assumption of a degree of freedom by deformation does not suffice for obtaining the desired motion. Indeed, it is possible to show that if the deformation of the plate depends upon a single essential parameter, the motion of the plate is a unique function of this parameter. But in this case, the plate cannot be shifted infinitely far by periodic deformations of its shape. The basic assumption which was formulated in the note is correct if greater freedom in the deformation of the plate is assumed or if the viscosity of the medium is introduced.

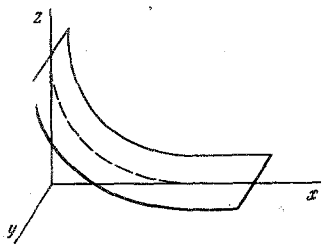

Fig. 1

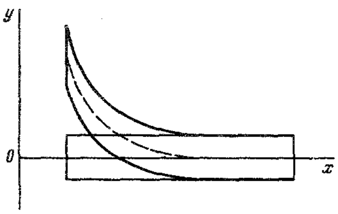

Fig. 2

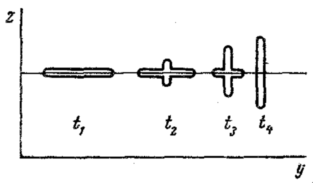

Fig. 3

Let us consider the mechanism of the motion of the model (plate) in an ideal liquid with the above-indicated additional assumption.

At the beginning, the body is at rest in the liquid. The axis of the body (of the plate) is situated in the xz plane and the curvature of the axis varies according to a linear law as a function of its length. Assume that the curvature at the end of the axis is zero and that the curvature increases with decreasing x (Fig. 1). Stresses which are proportional to the curvature are created in the plate. The stresses are conserved until the axis of the plate is straight. The potential energy of the stress is transformed into kinetic energy of the plane plate. The liquid also acquires some motion. Depending upon the parameters of the plate and the liquid, the plate runs over a certain path and stops (or almost stops). Is it then possible to transfer the plate into its initial shape without substantial shifting of the body? It was previously assumed that this is possible by introducing a small stress, and that the body can be transferred into its initial shape with a small displacement. However, in the scheme of an ideal liquid, the body then moves to the left and we obtain not only the initial shape but also the initial position of the body, i.e., no movement results.

The body can return into its initial shape almost without changing its position when the following deformations occur.

The first deformation, in which the xy plane is conserved as the plane of the plate's symmetry, transforms the plate so that the axis of the plate is converted into a line coinciding with the initial center line of the plate (Fig. 2).

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The resulting plate is subjected to a new deformation such that each cross section of the plate in the plane x = const is converted into the same cross section perpendicular to the former cross section (Fig. 3).

The second deformation results in an almost nonmoving body with the initial characteristics (rotated by 90° around the x axis). By repeating this process, the body can move forever with "any" velocity by virtue of internal forces; no singular points are generated in the liquid.

The same conclusions can be obtained under the conditions described in the aforementioned note in the direct deformation of the plane plate into its initial shape, provided that we introduce the viscosity of the medium: the viscosity must increase with increasing deformation rate.

These qualitative concepts can be developed in various directions, taking into account the viscosity and the singularities which develop in the liquid. It is interesting to estimate the role of eddies in the drag produced by the eddy shroud when various initial conditions and the above scheme of bending deformations are employed. It is interesting to determine the most advantageous shape of swimming bodies and the laws governing their deformations.